Towards Efficient Modularity in Drying: A Combinatorial Optimization Viewpoint

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CENTER FOR ADVANCED RESEARCH IN DRYING

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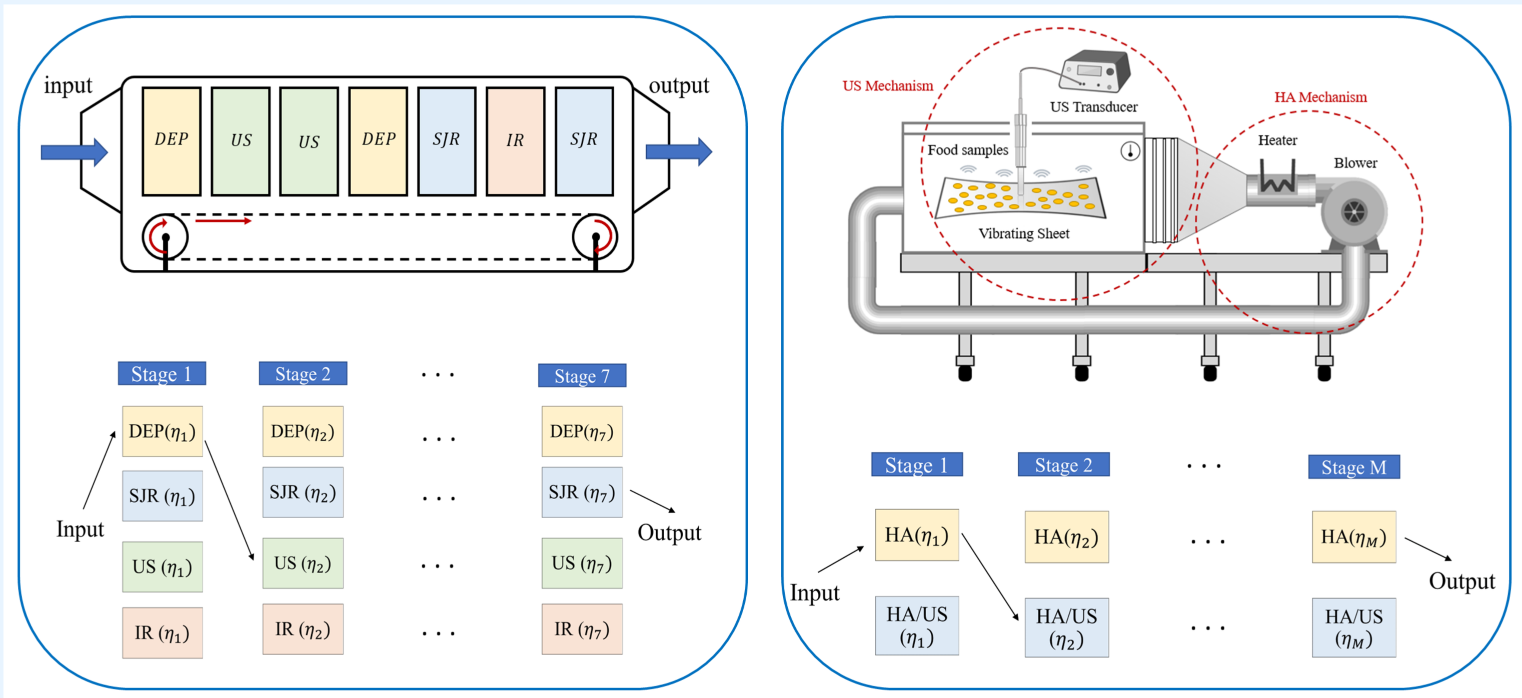
Motivation

- Industrial drying accounts for a significant proportion of energy consumption in manufacturing (12% or 1.2 quads/year), and adopting efficient controls and drying technologies could reduce this by up to 40% (0.5 quads/year), resulting in cost savings of up to $8 billion per year while also impacting food product quality
- Adopting a modular approach to industrial drying by combining multiple drying technologies with optimal sequencing and control parameters for each can lead to cost-efficient and high-performing drying
- The problem of *simultaneously* determining the optimal a) process configuration and b) operating conditions of each sub-process is NP-hard, with a non-convex cost surface containing multiple poor local minima



Problem Formulation

General Formulation

$$\min_{\{\eta_k\},v(\omega)} \sum_{\omega\in\Omega} v(\omega)D(\omega,\eta_1,\dots,\eta_M),$$
$$\text{subject to: } \sum_{\omega\in\Omega} v(\omega)=1,\ \ v(\omega)\in\{0,1\},\ \ \eta_k\in H(\gamma_k)\quad \forall 1\le k\le M,$$

- $\omega := (\gamma_1,\gamma_2,\dots,\gamma_M),\ \gamma_k\in\Gamma_k$: Sub-processes' sequence
- $\Gamma_k := \{f_{k1},\dots,f_{kL_k}\}$: Set of permissible sub-processes for k−th stage
- $\eta_k\in H(\gamma_k)$: Control parameters of the k−th sub-process

UIUC Hot-Air/Ultrasound Batch-process Testbed

- Decision Variables:

$$\gamma_k=\begin{cases}1 & \text{HA/US}\\ 0 & \text{HA}\end{cases},\ \ \eta_k=\begin{bmatrix}t_k\\T_k\end{bmatrix}\in U:=\left\{\begin{bmatrix}t\\T\end{bmatrix}\in\mathbb{R}^2: t\ge 2, T\in[30,70]^\circ C\right\}$$

- Dynamics: Moisture content kinetics

$$x_k^{(\omega)}=f_{\gamma_k}(x_{k-1}^{(\omega)},\eta_k),\quad x_0^{(\omega)}:\ \text{initial moisture content}$$

- Process Cost: Energy consumption + Terminal moisture content cost

$$D(\omega,\eta_1,\dots,\eta_M)=\sum_{k=1}^{M}(\underbrace{\alpha\dot m_{air}c_p(T_k-T_0)}_{\text{HA}}+\underbrace{\gamma_k P_{US}}_{\text{US}})t_k+\underbrace{G(x_M^{(\omega)},x_d)}_{\text{Terminal Penalty}}$$

Problem solution

- Maximum entropy principle (MEP):

$$\max_{p(\omega)} H:=-\sum_{\omega\in\Omega}p(\omega)\log(p(\omega))$$
$$\text{subject to: } \bar D:=\sum_{\omega\in\Omega}p(\omega)D(\omega,\eta_1,\dots,\eta_M)=D_0,\quad \sum_{\omega\in\Omega}p(\omega)=1 \qquad (1)$$

- Free energy (F): The Lagrangian of the above problem

$$F:=\bar D-\frac{1}{\beta}H+\mu\Big(\sum_{\omega\in\Omega}p(\omega)-1\Big)$$
$$\begin{cases}\beta\to 0: & H \text{ dominates} \implies F \text{ is convex}\\ \beta\to\infty: & D \text{ dominates} \implies F\approx D\end{cases}$$

The most unbiased probability distribution over the space of all sequences solves (1), which is obtained by Gibbs' distribution.

$$\frac{\partial F}{\partial p}=0 \implies p^*(\omega)=\frac{\exp(-\beta D(\omega,\eta_1,\dots,\eta_M))}{\sum_{\omega'\in\Omega}\exp(-\beta D(\omega',\eta_1,\dots,\eta_M))}$$
$$\implies F^*=\min_{p(\omega)}F=-\frac{1}{\beta}\log\sum_{\omega\in\Omega}\exp(-\beta D(\omega,\eta_1,\dots,\eta_M))$$

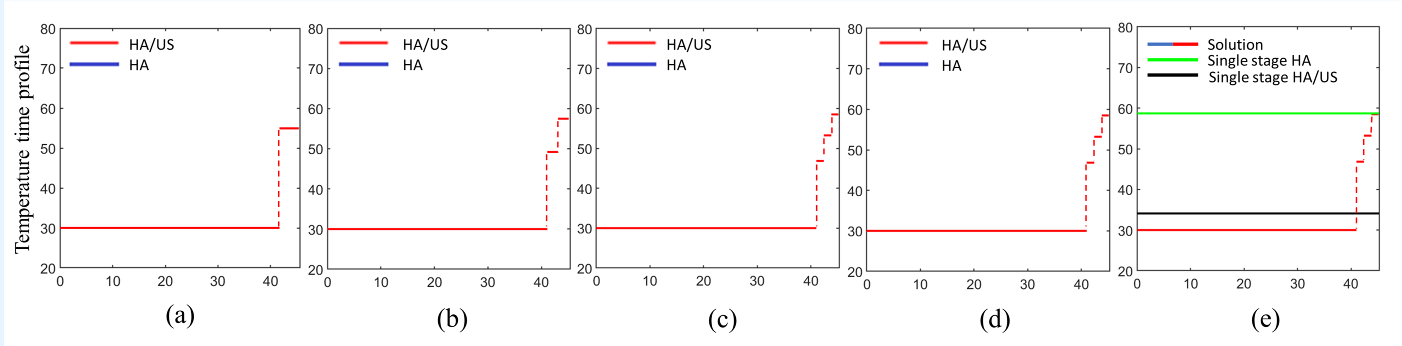
Thus, the following are the three main steps of the algorithm:

1. Updating path probabilities: $p(\omega)\leftarrow p^*(\omega)\quad \forall\omega\in\Omega$
2. Updating control parameters: $\{\eta_k\}\leftarrow \arg\min_{\{\eta_k\in U\}}F^*$
3. Increase β

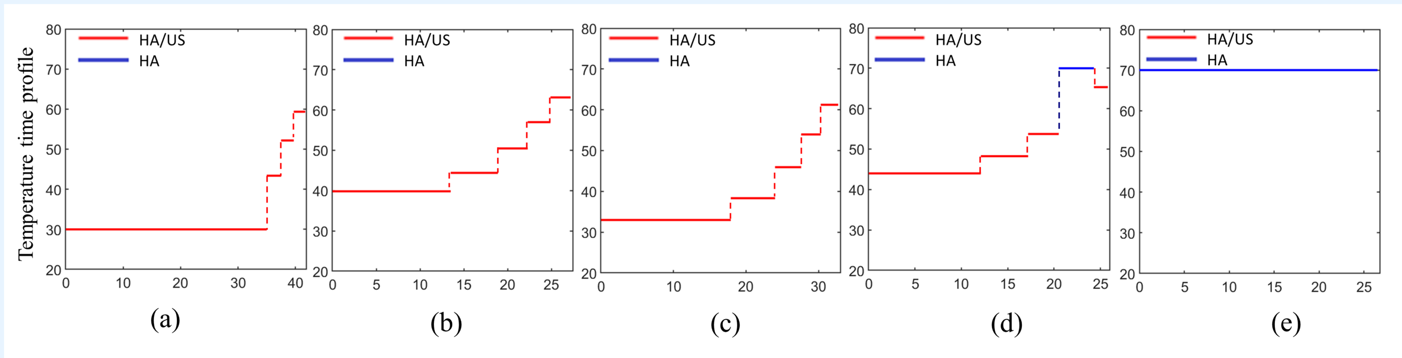
- $\lim_{\beta\to\infty}p(\omega)$ converges to zero for all non-optimal paths, while it converges to one for the optimal path

Results

- Varying the permissible number of stages (M): Results are shown for $M=2,3,4,5$. For $M\ge 4$, the total energy consumption is reduced by 63.19% compared to the optimal single-stage HA, and 12.09% compared to the optimal single-stage HA/US. Going beyond $M=4$ does not significantly affect the cost.

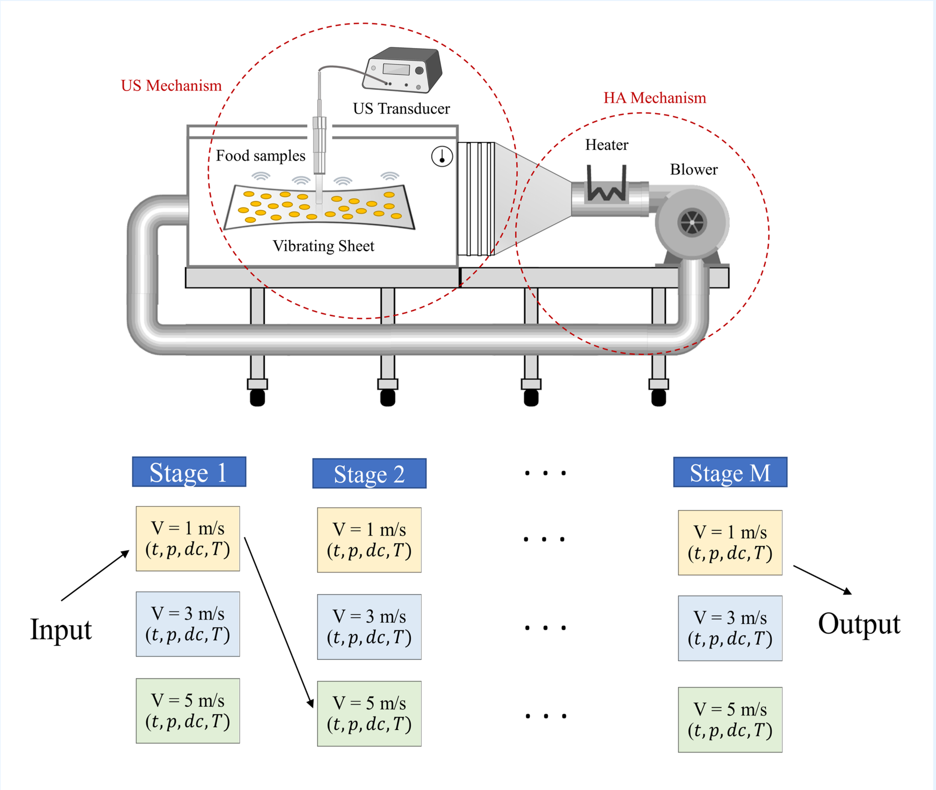


- Varying the relative weight of HA process (α): Decreasing α from 0.2 to 0.04 → Increase the cost on the US mechanism. The solutions achieved 9.95%, 5.75%, 3.62%, and 2.08% improvement compared to the most efficient single-stage processes, respectively.

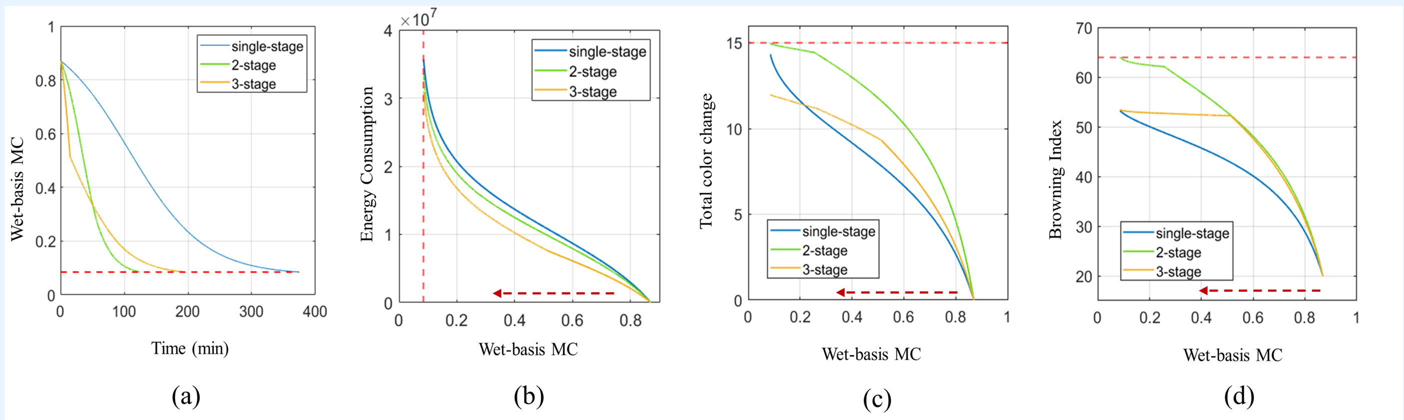


Extension to Mixed-integer Dynamic Optimization

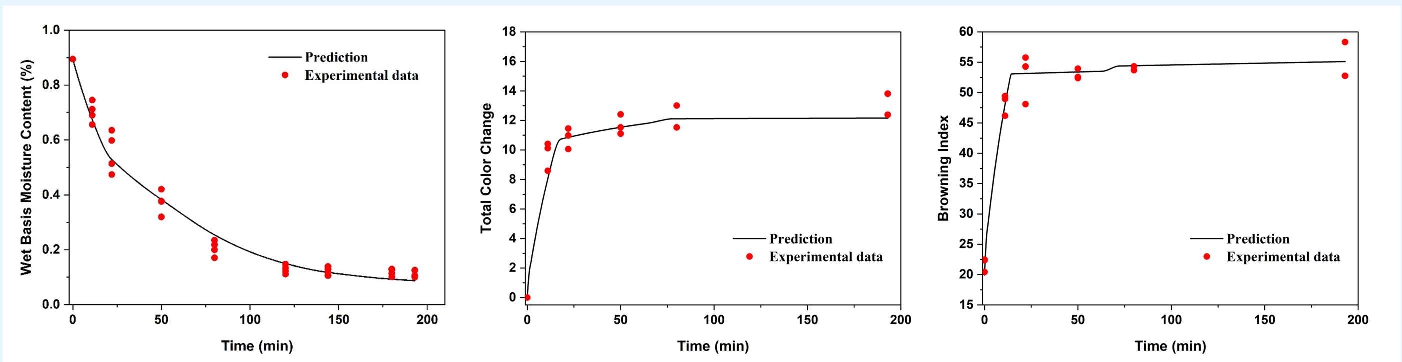
- Problem formulation: 1) Air velocity (V) is discrete, 2) Air temperature (T), US power (p), and US duty cycle (dc) are continuous.



- Results: 13.2% improvement compared to the optimal single-stage process that satisfies final product constraints on moisture content, total color change, and browning index.



- Validation: Validating the optimal solution with actual experiments.



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